TARGETED COLLEGE ADMISSION

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Abstract. This paper studies the *targeted college admission problem*, where students are matched to schools and firms simultaneously. We analyze conditions for stable matchings, demonstrating existence under structured preferences (e.g., homogeneous firm-school rankings or fixed capacities) and providing counterexamples where stability fails. An adapted deferred acceptance algorithm is proposed for cases where stability is guaranteed.

Keywords: targeted admission, college admission, three-sided matching, market design

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1. Introduction

This work investigates the targeted college admission problem, a three-sided matching problem where students must be assigned to both schools and firms simultaneously. Unlike classical two-sided stable matching (e.g., the Gale-Shapley model (1)), this setting introduces additional complexity due to interactions between schools, firms, and students.

The current system of targeted admissions in Russia creates inefficiencies due to mismatched regulations. Students must commit to a single employer-sponsored program before knowing their admission chances, while employers cannot control how many students they ultimately enroll. This leads to uncertainty for both sides. Our model resolves these issues by: *allowing students* to rank multiple sponsored programs, aligning with their true preferences. And *letting employers* specify desired enrollment limits, ensuring predictable

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outcomes. This ensures fairer, more efficient matching between students, universities, and employers.

We identify two key properties. The first concerns the preferences of firms. Preferences are called **homogeneous** if they follow the structure: $P_{f|s} = P_s$, meaning that firms rank schools and, within each school, replicate their preferences over students. Preferences may additionally be **sparse** — firms may reject some students, but the ranking order within schools remains unchanged.

The second property involves firms **capacities** (q_f). A capacity is **fixed** if a firm and a school pre-negotiate the number of slots allocated to students from that school. Otherwise, the capacity is **flexible**: for instance, a firm needs to hire 10 students, but their distribution across programs is unspecified in advance.

First of all, we demonstrate that stable matchings may fail to exist in the general case of our model (flexible capacity and heterogeneous preferences). To prove this, we adapt the construction by Boros et al. (2) to our targeted admission setting.

Let $|I| = |S| = |F| = 3$, with quotas $q_{s_i} = 1$ and $q_{f_i} = 1$ for all $i = 1, 2, 3$. The preferences follow a *lexicographic relaxation of cyclic preferences*: Firms F rank schools first (then students), schools S only care about students, students I rank firms first (then schools).

Explicitly, the preferences are given by:

- **Firms (F)**:

$$\begin{aligned} f_t &: (s_1, i_i) \succ (s_2, i_j) \succ (s_3, i_k) \quad \text{for } t \in \{1, 3\}, \\ f_2 &: (s_2, i_i) \succ (s_3, i_j) \succ (s_1, i_k), \\ f_t &: (s_i, i_2) \succ (s_i, i_1) \succ (s_i, i_3) \quad \text{for } t \in \{1, 2, 3\}. \end{aligned}$$

  And then comes all the other preferences.

- **Students (I)**:

$$\begin{aligned} i &: (f_1, s_i) \succ (f_2, s_j) \succ (f_3, s_k) \quad \text{for } i \in \{i_2, i_3\}, \\ i_1 &: (f_2, s_i) \succ (f_3, s_j) \succ (f_1, s_k), \\ i &: (f_i, s_2) \succ (f_i, s_1) \succ (f_i, s_3) \quad \text{for } i \in \{i_1, i_2, i_3\}. \end{aligned}$$

- **Schools** (S):

$$s : (i_1, \cdot) \succ (i_2, \cdot) \succ (i_3, \cdot) \quad \text{for } s \in \{s_1, s_3\},$$
$$s_2 : (i_2, \cdot) \succ (i_3, \cdot) \succ (i_1, \cdot).$$

We demonstrate that when firm-school capacities are fixed, the problem reduces to a two-sided matching scenario, where firms and schools can be treated as a single combined entity. Using a deferred acceptance algorithm with student-proposing dynamics, we prove the existence of a stable matching in this setting.

Furthermore, we analyze realistic special cases, such as when all schools rank students identically (e.g., based on standardized exam scores). This structure ensures stability while maintaining practical applicability in centralized admission systems.

When capacities are flexible, we hypothesize that stability can be achieved under mild preference restrictions - notably, homogeneous preferences across firms. To prove this, we employ matching-with-contracts theory, extending the model to accommodate multi-dimensional constraints while preserving stability conditions.

References

[1] Gale and Shapley, College Admissions and the Stability of Marriage [CrossRef]

[2] Boros, Gurvich et.al., Stable matchings in three-sided systems with cyclic preferences[CrossRef]