UDC 519.853.62
Zero-order optimization with Markovian Noise

S. Chebykin, A. Beznosikov
Moscow Institute of Physics and Technology

May 17, 2024

Abstract

In this work we study upper bounds for optimal convergence rates in stochastic optimization with smooth strongly convex objective function and zero-order unbiased oracle with bounded markovian noise. We adapt randomized accelerated GD to zero-order one-point oracle and provide argument and function convergence rates matching best known non-markovian ones.

1 Problem Statement

We study the minimization problem

$$\min_{x \in \mathbb{R}^d} f(x) := \mathbb{E}_{Z \sim \pi}[F(x, Z)]$$

where the access to the function f is available only through the unbiased noisy oracle $F(x, Z)$ and we cannot request the function value at a single point on more than one sample of Z (one-point feedback).

Our research aims to expand the results of [2] by adapting their algorithm, which successfully copes with markovian noise, to zero-order only oracle. Following the article, we consider additional assumptions:

- (A1,A2) f is (μ, L)-smooth.
- (A3) Z is uniformly geometrically ergodic with mixing time τ.
- (A4) $F(x, Z) = f(x) + h(x, Z)$ and

$$|h(x, Z)|^2 < \sigma^2$$

To estimate gradient we use finite difference:

$$\nabla f \approx g = d\frac{F(x + te, Z_1) - F(x - te, Z_2)}{2t} e, e \sim S^d(1)$$

And for function convergence we employ a well-known observation [1]

$$E_e g = \nabla E_e f(x + te)$$

2 Results

The main result of our work is the following recurrents.
Theorem 1.

$$E\left[\mu\|x^{k+1}-x^*\|^2+6(f(x_f^{k+1})-f(x^*))\right]$$
$$\leq\left(1-\sqrt{\frac{p_k^2\mu\gamma_k}{3}}\right)\left[\mu\|x^k-x^*\|^2+6(f(x_f^k)-f(x^*))\right]$$
$$+\frac{6\cdot 46p_k^2\gamma_k d^2C_1\tau}{t_k^2b}\sigma^2$$
$$+3L^2t_k^2\gamma_k p_k\left(1+6p_kd^2\right)$$
$$+12Lt_k^2$$

for function convergence

and

$$E\left[\|x^{k+1}-x^*\|^2+\frac{6}{\mu}(f(x_f^{k+1})-f(x^*))\right]$$
$$\leq\left(1-\sqrt{\frac{p_k^2\mu\gamma_k}{3}}\right)\left[\|x^k-x^*\|^2+\frac{6}{\mu}(f(x_f^k)-f(x^*))\right]$$
$$+\frac{3\cdot 92p_k\gamma_k d^2C_1\tau\sigma^2}{\mu t_k^2b}+\frac{21L^2t_k^2\gamma_k d^2}{\mu}+\frac{\sqrt{3}L^2t_k^2\sqrt{\gamma_k}d^2}{\sqrt{\mu}}$$

for argument convergence

Note that while t_k can be easily optimized away, setting the optimal γ_k is nontrivial.

3 References

[1] Arya Akhavan, Massimiliano Pontil, and Alexandre B. Tsybakov. “Exploiting Higher Order Smoothness in Derivative-free Optimization and Continuous Bandits”. In: *CoRR* abs/2006.07862 (2020). arXiv: `2006.07862`. URL: `https://arxiv.org/abs/2006.07862`.

[2] Aleksandr Beznosikov et al. *First Order Methods with Markovian Noise: from Acceleration to Variational Inequalities*. 2023. arXiv: `2305.15938 [math.OC]`.