Influence of hyperparameters on aggregating predictions of infinite number of experts

Kunin-Bogoiavlenskii Sergey
Expert: R. D. Zukhba
Consultant: A. V. Zukhba

Moscow Institute of Physics and Technology

kunin-bogoiavlenskii.sm@phystech.edu

April 2, 2024

Contents

What can be forecast?

> Tell us what the future holds, so we may know that you are gods.
>
> *Isaiah 41:23*

- Weather conditions
- Economic trends
- Technology advancements
- Consumer behavior
- Population growth
- Political elections outcomes

Targets

> Prediction is very difficult,
> especially if it's about the future.
>
> *Niels Bohr*

1. Time series generator implementation
2. Aggregating algorithm implementation
3. Experiments with various hyperparameters

Problem statement

> There are two kinds of forecasters: those who don't know, and those who don't know they don't know.
>
> *John Kenneth Galbraith*

Data

It is assumed that there are multiple generators, whose structure is unknown to the predictors. These generators switch, producing a time series that is subdivided into a sequence of segments - areas of stationarity, which can be studied using machine learning methods.

Gerators implemented:

- Linear
- ARMA

Problem statement

Terms

- X — signals space
- Y — responses space
- $\mathcal{N}$ — set of experts, indexed by natural numbers
- D — desicion space, to which predictions belong
- $\lambda : \mathrm{D} \times \mathrm{Y} \to \mathbb{R}_+$ — nonnegative loss function
- $L^i_T = \sum\limits_{t=1}^{T} l^i_t$ — cumulative loss of expert i during the first T steps
- $H_T = \sum\limits_{t=1}^{T} h_t$ — master's cumulative loss during the first T steps
- $R^i_T = H_T - L^i_T$ — master's regret relative to the expert i

Problem statement

Algorithm

FOR $t = 1, 2, \dots :$

1. Expert f^t initialization
2. Experts' predictions $f_t^i = f_t^i(x_t),\ 1 \leq i \leq t$
3. Master's prediction evaluation $\gamma_t = \mathrm{Subst}(\mathbf{f_t}, \widehat{\mathbf{w}}_\mathbf{t})$
4. Computation of master's loss $h_t = \lambda(p_t, y_t)$ and experts' losses l_t^i
5. **Loss Update** weights modification
6. **Mixing Update** weights modification

ENDFOR

Experiments

Initialization weights

Default weights: $w_1^i = \frac{1}{(i+1)\ln^2(i+1)}$

Experimental: $\frac{1}{i^\alpha}$, $\frac{1}{(i+1)\ln(i+1)\ln^2\ln(i+1)}$, etc.

Window size

As the algorithm does not know the locations of generator switches, finding an optimal training window is also a challenge.

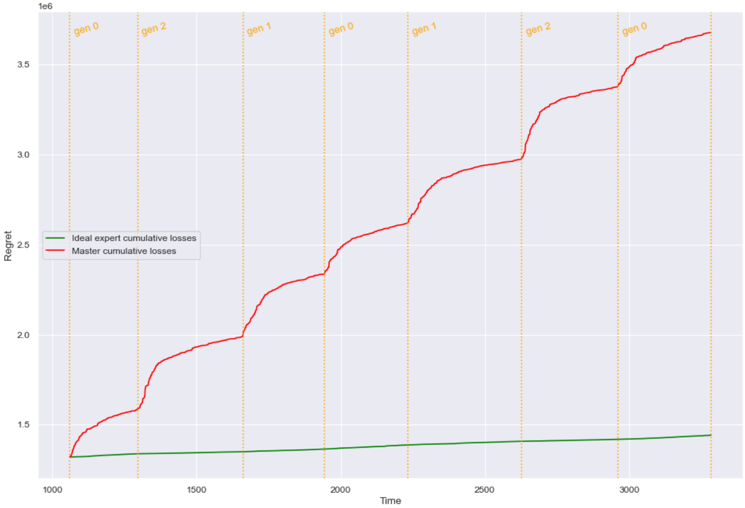
Mixing update coefficients

Default coefficient: $\alpha_t = \frac{1}{t+1}$

Experimental: $\frac{1}{(t+1)^\beta}$, etc.

Metric - the regret

Losses plot


1e6
gen 0
gen 2
gen 1
gen 0
gen 1
gen 2
gen 0
3.5
3.0
2.5
2.0
1.5
Regret
Ideal expert cumulative losses
Master cumulative losses
1000
1500
2000
2500
3000
Time

The End